

# Heavy Quark Symmetry in Isosinglet Nonleptonic $B$ -Decays

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## Abstract

We use a factorization theorem from the soft-collinear effective theory along with heavy quark symmetry to make model independent predictions for  $\bar{B}^0 \rightarrow D^{(*)0}M$  where  $M = \{\eta, \eta', \phi, \omega\}$ . Gluon production of these isosinglet mesons is included. We predict the equality of branching fractions in the  $\bar{B} \rightarrow DM$  and  $\bar{B} \rightarrow D^*M$  channels, with corrections at order  $\Lambda_{\text{QCD}}/Q$  and  $\alpha_s(Q)$  where  $Q = m_b, m_c$ , or  $E_M$ . We also predict that  $Br(\bar{B}^0 \rightarrow D\eta')/Br(\bar{B}^0 \rightarrow D\eta) = \tan^2 \theta = 0.67$  and  $Br(\bar{B} \rightarrow D\phi)/Br(\bar{B} \rightarrow D\omega) \lesssim 0.2$ , where here there are also  $\alpha_s(\sqrt{E\Lambda})$  corrections. These results agree well with the available data. A test for SU(3) violation in these decays is constructed.

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Nonleptonic weak-decays involving  $b \rightarrow c\bar{q}q'$  transitions provide an interesting framework for testing power expansions and factorization in QCD at the  $m_b \sim 5 \text{ GeV}$  scale. The Soft-Collinear Effective Theory (SCET) [1, 2, 3, 4] has been used to make predictions for two-body non-leptonic  $b \rightarrow c$  decays such as color allowed decays  $\bar{B} \rightarrow D^{(*)}M^-$  where  $M = \pi, \rho, K, K^*$  [5], color suppressed decays  $\bar{B}^0 \rightarrow D^{(*)0}M^0$  [6], decays to excited  $D$  mesons  $\bar{B} \rightarrow D^{**}M$  [7], as well as baryon decays  $\Lambda_b \rightarrow \Lambda_c M$  and  $\Lambda_b \rightarrow \Sigma_c^{(*)}M$  [8]. These predictions make use of a systematic expansion in  $\Lambda_{\text{QCD}}/m_{b,c}$  and  $\Lambda_{\text{QCD}}/E_M$ . For earlier work on color allowed decays see [9, 10, 11, 12]. The nature of factorization has also been studied in inclusive  $B \rightarrow D^{(*)}X$  decays, as well as decays to multi-body final states like  $B \rightarrow D\pi\pi\pi\pi$ , and decays to higher spin mesons [13, 14, 15, 16].

The Belle and BaBar Collaborations have recently reported measurements of the color suppressed decay channels  $\bar{B}^0 \rightarrow D^{(*)0}\eta$ ,  $\bar{B}^0 \rightarrow D^0\eta'$ , and  $\bar{B}^0 \rightarrow D^{(*)0}\omega$  which have an isosinglet meson  $M$  in the final state [17, 18, 19]. A summary of the data is given in Table I. By now it is well understood that “naive” factorization [20] fails miserably for these “color-suppressed” decays. A rigorous framework for discussing them in QCD is provided by the factorization theorem derived in Ref. [6]. The presence of isosinglet mesons enriches the structure of the decays due to  $\eta$ - $\eta'$  and  $\omega$ - $\phi$  mixing effects and gluon production mechanisms [21, 22, 23]. In this paper, we generalize the SCET analysis of [6] to include isosinglets. We also construct a test of SU(3) flavor symmetry in color suppressed decays, using our results to include the  $\eta - \eta'$  mixing.

The quark level weak Hamiltonian is

$$\mathcal{H}_W = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* [C_1(\mu)(\bar{c}b)_{V-A}(\bar{d}u)_{V-A} + C_2(\mu)(\bar{c}_i b_j)_{V-A}(\bar{d}_j u_i)_{V-A}], \quad (1)$$

where  $C_1$  and  $C_2$  are Wilson coefficients. For color-suppressed decay channels it gives rise to three flavor amplitudes denoted  $C$ ,  $E$ , and  $G$  in Fig. 1, which take on a precise meaning in terms of operators in the SCET analysis at leading order in  $\Lambda_{\text{QCD}}/Q$ . Here  $Q$  is a hard scale

Decay	Br( $10^{-4}$ ) (BaBar)	Br( $10^{-4}$ ) (Belle)	Br( $10^{-4}$ ) (Avg.)	$ A $ ( $10^{-4}$ MeV)
$\bar{B}^0 \rightarrow D^0\eta$	$2.5 \pm 0.2 \pm 0.3$	$1.83 \pm 0.15 \pm 0.27$	$2.1 \pm 0.2$	$1.67 \pm 0.09$
$\bar{B}^0 \rightarrow D^{*0}\eta$	$2.6 \pm 0.4 \pm 0.4$	—	$2.6 \pm 0.6$	$1.87 \pm 0.22$
$\bar{B}^0 \rightarrow D^0\eta'$	$1.7 \pm 0.4 \pm 0.2$	$1.14 \pm 0.20 \pm 0.11$	$1.3 \pm 0.2$	$1.31 \pm 0.11$
$\bar{B}^0 \rightarrow D^{*0}\eta'$	$1.3 \pm 0.7 \pm 0.2$	$1.26 \pm 0.35 \pm 0.25$	$1.3 \pm 0.4$	$1.33 \pm 0.19$
$\bar{B}^0 \rightarrow D^0\omega$	$3.0 \pm 0.3 \pm 0.4$	$2.25 \pm 0.21 \pm 0.28$	$2.5 \pm 0.3$	$1.83 \pm 0.11$
$\bar{B}^0 \rightarrow D^{*0}\omega$	$4.2 \pm 0.7 \pm 0.9$	—	$4.2 \pm 1.1$	$2.40 \pm 0.31$
$\bar{B}^0 \rightarrow D^{(*)0}\phi$	—	—	—	—
$\bar{B}^0 \rightarrow D^0\pi^0$	$2.9 \pm 0.2 \pm 0.3$	$2.31 \pm 0.12 \pm 0.23$	$2.5 \pm 0.2$	$1.81 \pm 0.08$
$\bar{B}^0 \rightarrow D^{*0}\pi^0$	—	—	$2.8 \pm 0.5$	$1.95 \pm 0.18$
$\bar{B}^0 \rightarrow D^0\bar{K}^0$	$0.62 \pm 0.12 \pm 0.04$	$0.50^{+0.13}_{-0.12} \pm 0.06$	$0.44 \pm 0.06$	$0.76 \pm 0.06$
$\bar{B}^0 \rightarrow D^{*0}\bar{K}^0$	$0.45 \pm 0.19 \pm 0.05$	$< 0.66$	$0.36 \pm 0.10$	$0.69 \pm 0.10$
$\bar{B}^0 \rightarrow D_s^+ K^-$	$0.32 \pm 0.10 \pm 0.10$	$0.293 \pm 0.055 \pm 0.079$	$0.30 \pm 0.08$	$0.64 \pm 0.08$

TABLE I: Data on  $B \rightarrow D$  and  $B \rightarrow D^*$  decays with isosinglet light mesons and the weighted average. The BaBar data is from Ref. [17] and the Belle data is from Refs. [18] and [24].

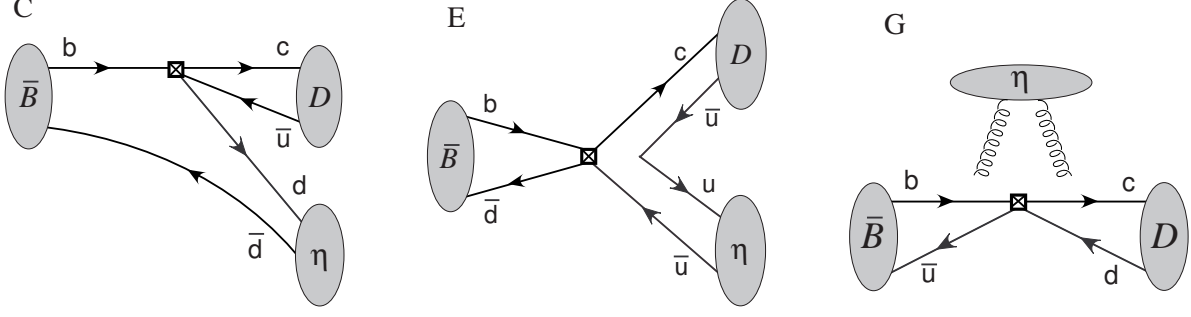


FIG. 1: Flavor diagrams for  $\bar{B} \rightarrow D\eta$  decays, referred to as color-suppressed (C),  $W$ -exchange (E), and gluon production (G). These amplitudes denote classes of Feynman diagrams where the remaining terms in a class are generated by adding any number of gluons as well as light-quark loops to the pictures.

on the order of the heavy quark masses  $m_b, m_c$  or the isosinglet meson energy  $E_M$ . The gluon  $G$  amplitude is unique to isosinglet mesons. We will show however that for  $B \rightarrow D^{(*)}M$  decays the  $G$  amplitude is suppressed by  $\alpha_s(\sqrt{E\Lambda})$  relative to the  $C, E$  contributions.

For color suppressed decays to isosinglet mesons  $M = \{\eta, \eta', \omega, \phi\}$  we will show that the factorization theorem for the amplitudes  $A_M^{(*)} = \langle D^{(*)}M | H_W | \bar{B}^0 \rangle$  is

$$A^{(*)M} = A_{\text{short}}^{(*)M} + A_{\text{glue}}^{(*)M} + A_{\text{long}}^{(*)M} \pm (L \leftrightarrow R), \quad (2)$$

where the  $\pm$  refers to the cases  $DM, D^*M$  and the three amplitudes at LO are

$$\begin{aligned} A_{\text{short}}^{(*)M} &= N_q^M \sum_{i=0,8} \int_0^1 dx dz \int dk_1^+ dk_2^+ C_L^{(i)}(z) J_q^{(i)}(z, x, k_1^+, k_2^+) S_L^{(i)}(k_1^+, k_2^+) \phi_q^M(x), \\ A_{\text{glue}}^{(*)M} &= N_g^M \sum_{i=0,8} \int_0^1 dx dz \int dk_1^+ dk_2^+ C_L^{(i)}(z) J_g^{(i)}(z, x, k_1^+, k_2^+) S_L^{(i)}(k_1^+, k_2^+) \bar{\phi}_g^M(x), \\ A_{\text{long}}^{(*)M} &= N_q^M \sum_{i=0,8} \int_0^1 dz \int dk^+ d\omega \int d^2x_\perp C_L^{(i)}(z) \bar{J}^{(i)}(\omega k^+) \Phi_L^{(i)}(k^+, x_\perp, \varepsilon_{D^*}^*) \Psi_M^{(i)}(z, \omega, x_\perp, \varepsilon_M^*), \end{aligned} \quad (3)$$

where  $i = 0, 8$  are for two different color structures. Here  $A_{\text{short}}^{(*)M}$  and  $A_{\text{long}}^{(*)M}$  are very similar to the results derived for non-singlet mesons in Ref. [6], and each contains a flavor-singlet subset of the sum of  $C$  and  $E$  graphs. The amplitude  $A_{\text{glue}}^{(*)M}$  contains the additional gluon contributions. The  $S_L^{(0,8)}$  are universal generalized distribution functions for the  $B \rightarrow D^{(*)}$  transition. The  $\phi_{q,g}^M$  are leading twist meson distribution functions, and <sup>1</sup>

$$N_q^M = \frac{1}{2} f_q^M G_F V_{cb} V_{ud}^* \sqrt{m_B m_{D^{(*)}}}, \quad N_g^M = \sqrt{\frac{8}{3}} f_1^M G_F V_{cb} V_{ud}^* \sqrt{m_B m_{D^{(*)}}}. \quad (4)$$

The  $\Phi_L^{(i)}$  and  $\Psi_M^{(i)}$  are long distance analogs of  $S_L^{(i)}$  and  $\phi^M$  where the  $x_\perp$  dependence does not factorize. At lowest order in the perturbative expansion,  $C_L^{(0)} = C_1 + C_2/3$  and  $C_L^{(8)} = 2C_2$

<sup>1</sup> For Cabibbo suppressed channels we replace  $V_{ud}^* \rightarrow V_{us}^*$  in  $N_q^M$  and  $N_g^M$ .

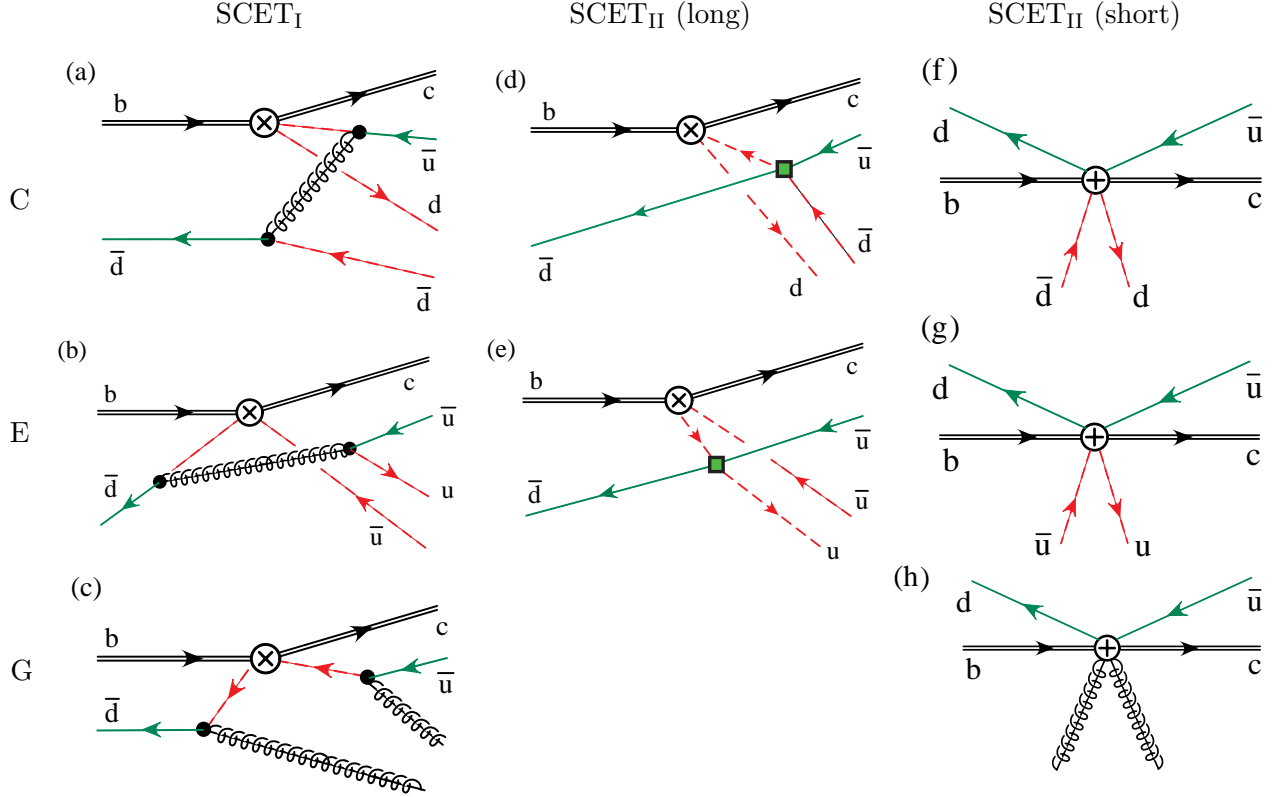


FIG. 2: Graphs for the tree level matching calculation from SCET<sub>I</sub> (a,b,c) onto SCET<sub>II</sub> (d,e,f,g,h). The dashed lines are collinear quark propagators and the spring with a line is a collinear gluon. Solid lines are quarks with momenta  $p^\mu \sim \Lambda$ . The  $\otimes$  denotes an insertion of the weak operator in the appropriate theory. The solid dots in (a,b,c) denote insertions of the mixed usoft-collinear quark action  $\mathcal{L}_{\xi q}^{(1)}$ . The boxes in (d,e) denote the SCET<sub>II</sub> operator  $\mathcal{L}_{\xi q q}^{(1)}$  from Ref. [6].

and are independent of the parameter  $z$ . The  $(L \leftrightarrow R)$  terms in Eq. (2) have small coefficients  $C_R^{(0,8)} \sim \mathcal{O}(\alpha_s(Q))$  and will be neglected in our phenomenological analysis. Finally, the jet functions  $J_q^{(i)}$ ,  $J_g^{(i)}$ , and  $\bar{J}^{(i)}$  are responsible for rearranging the quarks in the decay process; they can be computed in perturbation theory and are discussed further below.

The derivation of Eq. (3) involves subsequently integrating out the scales  $Q = \{m_b, m_c, E_M\}$  and then  $\sqrt{E_M \Lambda_{\text{QCD}}}$  by matching onto effective field theories,  $\text{QCD} \rightarrow \text{SCET}_I \rightarrow \text{SCET}_{II}$ , and we refer to Ref. [6] for notation and further details. Here we only give the reader a sense of the procedure, and discuss additions needed for the isosinglet case. In SCET<sub>I</sub> there is only a single time-ordered product for color suppressed decays

$$T_{L,R}^{(0,8)} = \frac{1}{2} \int d^4x d^4y T\{\mathcal{Q}_{L,R}^{(0,8)}(0), i\mathcal{L}_{\xi q}^{(1)}(x), i\mathcal{L}_{\xi q}^{(1)}(y)\}. \quad (5)$$

Here  $\mathcal{Q}_{L,R}^{(0,8)}(0)$  are the LO operators in SCET<sub>I</sub> that  $H_W$  gets matched onto, and  $\mathcal{L}_{\xi q}^{(1)}$  is the subleading ultrasoft-collinear interaction Lagrangian, which is the lowest order term that can change a ultrasoft quark into a collinear quark. The power suppression from the two  $\mathcal{L}_{\xi q}^{(1)}$ 's makes the amplitudes for color suppressed decays smaller by  $\Lambda/Q$  from those for color allowed decays. The  $C$ ,  $E$ , and  $G$  diagrams in Fig. 1 are different contractions of the terms in  $T_{L,R}^{(0,8)}$ , and at tree level are given by Figs. 2(a), 2(b), and 2(c) respectively. The propagators in these figures are offshell by  $p^2 \sim E_M \Lambda$ . In SCET<sub>II</sub> all lines are offshell by

$\sim \Lambda^2$ , so the propagators either collapse to a point as shown in Figs. 2(f), 2(g), and 2(h), or the quark propagator remains long distance as denoted in Figs. 2(d) and 2(e). For the terms in the factorization theorem in Eq. (3), Figs. 2(f,g) contribute to  $A_{\text{short}}$ , Fig. 2(h) contributes to  $A_{\text{glue}}$ , and Figs. 2(d,e) contributes to  $A_{\text{long}}$ . A notable feature is the absence of a long distance gluon contribution. Momentum conservation at the  $\mathcal{L}_{\xi q}^{(1)}$  vertex forbids the quark propagators in Fig. 2(c) from having a long distance component (or more generally there does not exist an appropriate analog of the shaded box operator in Figs. 2(d,e) that takes a soft  $\bar{d}$  to a soft  $\bar{u}$ ).

The diagrams in Fig. 2(f,g) have isosinglet and isotriplet components. The corresponding isosinglet operators in SCET<sub>II</sub> are [6]

$$\begin{aligned} O_j^{(0)}(k_i^+, \omega_k) &= \left[ \bar{h}_{v'}^{(c)} \Gamma_j^h h_v^{(b)} (\bar{d} S)_{k_1^+} \not{P}_L (S^\dagger u)_{k_2^+} \right] \left[ (\bar{\xi}_n^{(q)} W)_{\omega_1} \Gamma_c (W^\dagger \xi_n^{(q)})_{\omega_2} \right], \\ O_j^{(8)}(k_i^+, \omega_k) &= \left[ (\bar{h}_{v'}^{(c)} S) \Gamma_j^h T^a (S^\dagger h_v^{(b)}) (\bar{d} S)_{k_1^+} \not{P}_L T^a (S^\dagger u)_{k_2^+} \right] \left[ (\bar{\xi}_n^{(q)} W)_{\omega_1} \Gamma_c (W^\dagger \xi_n^{(q)})_{\omega_2} \right], \end{aligned} \quad (6)$$

where  $h_v$  and  $h_{v'}$  are Heavy Quark Effective Theory (HQET) fields for the bottom and charm quarks, the index  $j = L, R$  refers to the Dirac structures  $\Gamma_L^h = \not{P}_L$  or  $\Gamma_R^h = \not{P}_R$ ,  $\Gamma_c = (\not{P}_L)/2$ ,  $\xi_n^{(q)}$  are collinear quark fields and we sum over the  $q = u, d$  flavors. Note that no collinear strange quarks appear. In Eq. (6) the factors of  $W$  and  $S$  are Wilson lines required for gauge invariance and the momenta subscripts  $(\cdots)_{\omega_i}$  and  $(\cdots)_{k_i^+}$  refer to the momentum carried by the product of fields in the brackets. The matrix element of the soft fields in  $O_L^{(0,8)}$  gives the  $S_L^{(0,8)}(k_1^+, k_2^+)$  distribution functions, for example

$$\frac{\langle D^{(*)0}(v') | (\bar{h}_{v'}^{(c)} S) \not{P}_L (S^\dagger h_v^{(b)}) (\bar{d} S)_{k_1^+} \not{P}_L (S^\dagger u)_{k_2^+} | \bar{B}^0(v) \rangle}{\sqrt{m_B m_D}} = A^{D^{(*)}} S_L^{(0)}(k_1^+, k_2^+), \quad (7)$$

where  $A^D = 1$  and  $A^{D^*} = n \cdot \varepsilon^* / n \cdot v' = 1$  (since the polarization is longitudinal). The matrix element of the collinear operator gives the LO light-cone distribution functions. We work in the isospin limit and use the  $(u\bar{u} + d\bar{d})$ ,  $s\bar{s}$  basis for our quark operators. For  $M = \eta, \eta'$  we have

$$\begin{aligned} \langle M(p) | \sum_{q=u,d} (\bar{\xi}_n^{(q)} W)_{\omega_1} \frac{\not{\eta} \gamma_5}{\sqrt{2}} (W^\dagger \xi_n^{(q)})_{\omega_2} | 0 \rangle &= -i \bar{n} \cdot p f_q^M \phi_q^M(\mu, x), \\ \langle M(p) | (\bar{\xi}_n^{(s)} W)_{\omega_1} \not{\eta} \gamma_5 (W^\dagger \xi_n^{(s)})_{\omega_2} | 0 \rangle &= -i \bar{n} \cdot p f_s^M \phi_s^M(\mu, x), \end{aligned} \quad (8)$$

while for vector mesons  $M = \omega, \phi$  we simplify the dependence on the polarization using  $m_V \bar{n} \cdot \varepsilon^* = \bar{n} \cdot p$  and then have

$$\begin{aligned} \langle M(p, \varepsilon^*) | \sum_{q=u,d} (\bar{\xi}_n^{(q)} W)_{\omega_1} \frac{\not{\eta}}{\sqrt{2}} (W^\dagger \xi_n^{(q)})_{\omega_2} | 0 \rangle &= i \bar{n} \cdot p f_q^M \phi_q^M(\mu, x), \\ \langle M(p, \varepsilon^*) | (\bar{\xi}_n^{(s)} W)_{\omega_1} \not{\eta} (W^\dagger \xi_n^{(s)})_{\omega_2} | 0 \rangle &= i \bar{n} \cdot p f_s^M \phi_s^M(\mu, x). \end{aligned} \quad (9)$$

In both Eq. (8) and (9) we have suppressed a prefactor for the  $\phi^M$ 's on the RHS:

$$\int_0^1 dx \delta(\omega_1 - x \bar{n} \cdot p) \delta(\omega_2 + (1-x) \bar{n} \cdot p). \quad (10)$$

Note that these definitions make no assumption about  $\eta$ - $\eta'$  or  $\omega$ - $\phi$  mixing. The SCET operators in Eq. (6) only give rise to the  $\phi_q^M$  terms. By charge conjugation  $\phi_q^M(1-x) = \phi_q^M(x)$  and  $\phi_s^M(1-x) = \phi_s^M(x)$  for both the isosinglet pseudoscalars and isosinglet vectors. Our definitions agree with those in Ref. [22].

Now consider the graph emitting collinear gluons, Fig. 2(c). and integrate out the hard-collinear quark propagators to match onto Fig. 2(h). Writing the result of computing this Feynman diagram in terms of an operator gives a factor of  $[\bar{h}_{v'}^{(c)} \Gamma_j^h \{1, T^c\} h_v^{(b)}]$  times

$$[\bar{d} T^a \gamma_\perp^\mu P_L \{1, T^c\} \frac{\not{q}}{2} \gamma_\perp^\nu T^b u] (ig \mathcal{B}_\perp^{\mu a}) (ig \mathcal{B}_\perp^{\nu b}) \frac{-\bar{n} \cdot p_2}{-\bar{n} \cdot p_2 n \cdot k_2 + i\epsilon} \frac{\bar{n} \cdot p_1}{\bar{n} \cdot p_1 n \cdot k_1 + i\epsilon}, \quad (11)$$

where  $ig \mathcal{B}_{\perp\omega}^{\mu b} T^b = [1/\bar{\mathcal{P}} W^\dagger [i\bar{n} \cdot D_c, iD_{c\perp}^\mu] W]_\omega$  is a LO gauge invariant combination with the gluon field strength. The Dirac structure can be simplified:  $\gamma_\perp^\mu P_L \not{q} \gamma_\perp^\nu = -\not{q} P_L (g_\perp^{\mu\nu} + i\epsilon_\perp^{\mu\nu})$  where  $\epsilon_{12}^\perp = +1$ . Furthermore we only need to keep operators that are collinear color singlets, since others give vanishing contributions at this order. These simplifications hold at any order in perturbation theory in SCET<sub>I</sub>, so the matching gives only two SCET<sub>II</sub> operators

$$\begin{aligned} G_j^{(0)}(k_i^+, \omega_k) &= \left[ \bar{h}_{v'}^{(c)} \Gamma_j^h h_v^{(b)} (\bar{d} S)_{k_1^+} \not{q} P_L (S^\dagger u)_{k_2^+} \right] \left[ (g_{\mu\nu}^\perp + i\epsilon_{\mu\nu}^\perp) \mathcal{B}_{\perp\omega_1}^{\mu b} \mathcal{B}_{\perp\omega_2}^{\nu b} \right], \\ G_j^{(8)}(k_i^+, \omega_k) &= \left[ \bar{h}_{v'}^{(c)} \Gamma_j^h T^a h_v^{(b)} (\bar{d} S)_{k_1^+} \not{q} P_L T^a (S^\dagger u)_{k_2^+} \right] \left[ (g_{\mu\nu}^\perp + i\epsilon_{\mu\nu}^\perp) \mathcal{B}_{\perp\omega_1}^{\mu b} \mathcal{B}_{\perp\omega_2}^{\nu b} \right]. \end{aligned} \quad (12)$$

The operators in Eq. (12) appear as products of soft and collinear fields allowing us to factorize the amplitude into soft and collinear matrix elements. We immediately notice that the soft fields in Eq. (12) and Eq. (6) are identical. Thus, the same non-perturbative  $B \rightarrow D^{(*)}$  distribution functions  $S_L^{(0,8)}$  occur in the factorization theorem for the gluon and quark contributions (cf. Eq. (3)). The matrix elements of the collinear fields give

$$\begin{aligned} M = \eta, \eta' : \quad \langle M(p) | i\epsilon_{\mu\nu}^\perp \mathcal{B}_{\perp, -\omega_1}^{\mu b} \mathcal{B}_{\perp, \omega_2}^{\nu b} | 0 \rangle &= \frac{i}{2} \sqrt{C_F} f_1^M \bar{\phi}_M^g(\mu, x), \\ M = \phi, \omega : \quad \langle M(p) | g_{\mu\nu}^\perp \mathcal{B}_{\perp, -\omega_1}^{\mu b} \mathcal{B}_{\perp, \omega_2}^{\nu b} | 0 \rangle &= \frac{i}{2} \sqrt{C_F} f_1^M \bar{\phi}_M^g(\mu, x), \end{aligned} \quad (13)$$

where

$$\bar{\phi}_g^M(x, \mu) = \frac{\phi_g^M(x, \mu)}{x(1-x)}, \quad (14)$$

$C_F = (N_c^2 - 1)/(2N_c) = 4/3$ , and  $f_1^M = \sqrt{2/3} f_q^M + \sqrt{1/3} f_s^M$ . (We again suppressed a prefactor on the RHS of Eq. (13) which is given in Eq. (10).) Our  $\phi_g^\eta$  and  $\phi_g^{\eta'}$  are the same as the ones defined in Ref. [22], where they were used to analyze the  $\gamma$ - $\eta$  and  $\gamma$ - $\eta'$  form factors. Charge conjugation implies

$$\phi_g^M(1-x) = -\phi_g^M(x). \quad (15)$$

At tree level using Eq. (11) to match onto the gluon operators  $G_j^{(0,8)}$  gives

$$J_g^{(0)} = \frac{\pi \alpha_s(\mu_0)}{N_c(n \cdot k_2 - i\epsilon)(n \cdot k_1 + i\epsilon)}, \quad J_g^{(8)} = \frac{\pi \alpha_s(\mu_0)}{(-N_c^3 + N_c)(n \cdot k_2 - i\epsilon)(n \cdot k_1 + i\epsilon)}, \quad (16)$$

where more generally  $J_g^{(0,8)} = J_g^{(0,8)}(z, x, k_1^+, k_2^+)$ . Thus, the jet functions are even under  $x \rightarrow 1 - x$  while the gluon distributions are odd, and the convolution in Eq. (3) for  $A_{\text{glue}}^{(*)M}$  vanishes. Thus,  $A_{\text{glue}}^{(*)M}$  starts at  $\mathcal{O}[\alpha_s^2(\sqrt{E\Lambda})]$  from one-loop corrections to the gluon jet function.

The remaining contributions to the amplitude come from the isosinglet component of the long distance operators shown in Figs. 2(d,e). These operators take the form of a T-ordered product in SCET<sub>II</sub>

$$\overline{\mathcal{O}}_j^{(0,8)}(\omega_i, k^+, \omega, \mu) = \int d^4x T \mathcal{Q}_j^{(0,8)}(\omega_i, x=0) iL^{(0,8)}(\omega, k^+, x). \quad (17)$$

where  $L^{(0,8)}(\omega, k^+, x)$  [6] are four quark operators in SCET<sub>II</sub> denoted by the shaded boxes in Figs. 2(d,e). The matrix element of these long distance operators give the contribution  $A_{\text{long}}^{(*)M}$  in Eq. (3) where the collinear and soft functions  $\Psi_M^{(0,8)}$  and  $\Phi_L^{(0,8)}$  are defined as

$$\begin{aligned} & \left\langle M^0(p_M, \epsilon_M) \left| \left[ (\bar{\xi}_n^{(d)} W)_{\omega_1} \not{n} P_L (W^\dagger \xi_n^{(u)})_{\omega_2} \right] (0_\perp) \left[ (\bar{\xi}_n^{(u)} W)_\omega \not{n} P_L (W^\dagger \xi_n^{(d)})_\omega \right] (x_\perp) \right| 0 \right\rangle \\ &= i f^M / \sqrt{2} \Psi_M^{(0)}(z, \omega, x_\perp, \epsilon_M^*), \\ & \left\langle D^{(*)0}(v', \epsilon_{D^*}) \left| \left[ (\bar{h}_{v'}^{(c)} S) \not{n} P_L^h (S^\dagger h_v^{(b)}) \right] (0_\perp) \left[ (\bar{d} S)_{k^+} \not{n} P_L (S^\dagger u)_{k^+} \right] (x_\perp) \right| \bar{B}^0 \right\rangle \\ &= \sqrt{m_B m_{D^*}} \Phi_L^{(0)}(k^+, x_\perp, \epsilon_{D^*}^*), \end{aligned} \quad (18)$$

and at tree level the jet functions are  $\bar{J}^{(0)}(\omega k^+) = -4/3 \bar{J}^{(8)}(\omega k^+) = -8\pi\alpha_s(\mu)/(9\omega k^+)$ .

Eqs. (7,8,13,18) combined with Eq. (3) completely define the amplitude for color suppressed decays to leading nonvanishing order in  $\Lambda_{\text{QCD}}/Q$ . We are now in a position to make phenomenological predictions. We will neglect perturbative corrections at the hard scale,  $\alpha_s(Q)$ . For heavy quark symmetry predictions we will work to all orders in  $\alpha_s(\sqrt{E\Lambda})$ , while for relating the  $\eta$  and  $\eta'$  amplitudes we will work to leading order in  $\alpha_s(\sqrt{E\Lambda})$ .

The first class of predictions that we address make use of heavy quark symmetry to relate the  $D$  and  $D^*$  amplitudes. It is worth mentioning why such predictions are impossible to make using only HQET even though the  $D, D^*$  are in a symmetry multiplet. If we do not factorize the energetic pion out of the matrix element then the chromomagnetic operator which breaks the spin symmetry comes in with a factor of  $E_\pi/m_c \simeq 1.5$  and is not suppressed [25]. In the SCET analysis spin-symmetry breaking effects are guaranteed to be suppressed by  $\Lambda_{\text{QCD}}/m_c$  allowing for possible corrections at the  $\sim 25\%$  level.

The factorization theorem in SCET, Eq. (3), moves the energetic light meson into a separate matrix element. This allows us to use the formalism of HQET in the soft sector to relate the  $\bar{B} \rightarrow D$  and  $\bar{B} \rightarrow D^*$  matrix elements in Eqs. (7) and (18). For  $A_{\text{short}}^M$ , the contribution is the same for the  $D$  and  $D^*$  channels with identical soft functions  $S_L^{(i)}$  as a consequence of heavy quark symmetry. The same is true for the soft matrix element in  $A_{\text{glue}}$  which also gives  $S_L^{(i)}$ . For the long distance contribution  $A_{\text{long}}^M$ , in addition to a dependence on powers of  $x_\perp^2$ , the soft function  $\Phi_L^{(i)}(k^+, x_\perp, \epsilon_{D^*}^*)$  can have terms proportional to  $x_\perp \cdot \epsilon_{D^*}^*$  in the  $D^*$  channel while the collinear function  $\Psi_M^{(i)}(z, \omega, x_\perp, \epsilon_M^*)$  can have terms proportional to  $x_\perp \cdot \epsilon_M^*$  in the case of vector mesons. In the convolution over  $x_\perp$  in  $A_{\text{long}}^M$ , the term in the integrand proportional to the product  $(x_\perp \cdot \epsilon_{D^*}^*)(x_\perp \cdot \epsilon_M^*)$  can be non-vanishing in the  $D^*$  channel with a vector meson. Such terms do not appear in the  $D$  channel making the

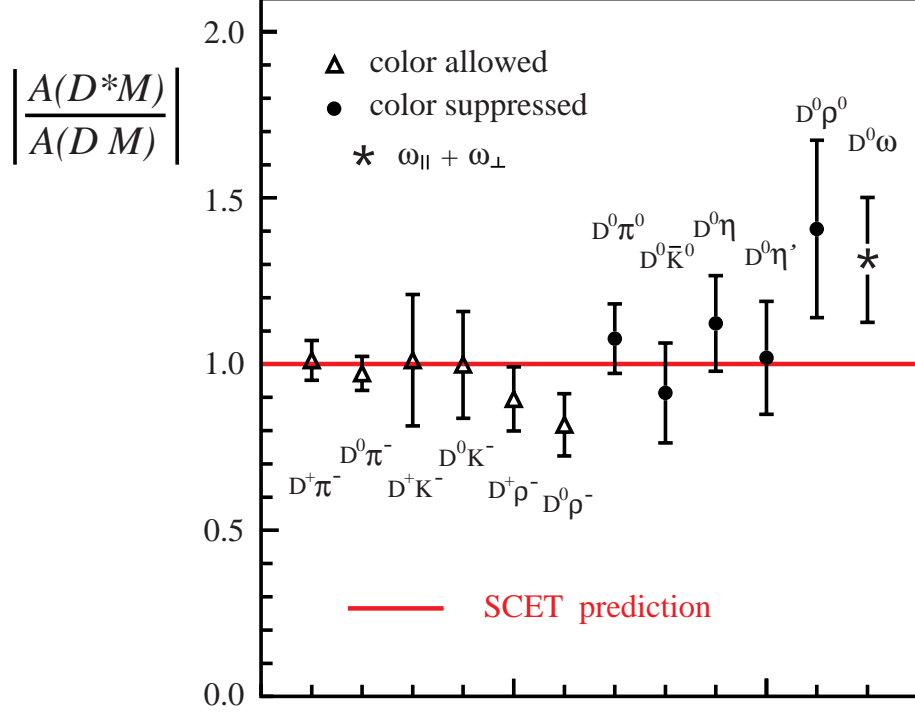


FIG. 3: Comparison of the absolute value of the ratio of the amplitude for  $B \rightarrow D^*M$  divided by the amplitude for  $B \rightarrow DM$  versus data from different channels. This ratio of amplitudes is predicted to be one at leading order in SCET. For  $\omega$ 's this prediction only holds for the longitudinal component, and the data shown is for longitudinal plus transverse.

$D$  and  $D^*$  amplitudes unrelated in general. However, if we restrict ourselves to longitudinal polarizations, such terms in the  $D^*$  channel vanish and the long distance contributions in the two channels become identical. Finally, note that the SCET<sub>I</sub> jet functions, and the other collinear matrix elements in SCET<sub>II</sub> are identical for the two channels. Thus, at leading order in  $\alpha_s(Q)$  and  $\Lambda_{\text{QCD}}/Q$  the  $D$  and  $D^*$  channels are related as

$$\frac{Br(\bar{B} \rightarrow D^*\eta)}{Br(\bar{B} \rightarrow D\eta)} = \frac{Br(\bar{B} \rightarrow D^*\eta')}{Br(\bar{B} \rightarrow D\eta')} = \frac{Br(\bar{B} \rightarrow D^*\omega_{\parallel})}{Br(\bar{B} \rightarrow D\omega)} = 1. \quad (19)$$

For the decay to  $\phi$ 's we also have

$$\frac{Br(\bar{B} \rightarrow D^*\phi_{\parallel})}{Br(\bar{B} \rightarrow D\phi)} = 1, \quad (20)$$

however in this case the prediction assumes that the  $\alpha_s^2(\sqrt{E\Lambda})$  contribution from  $A_{\text{glue}}$  dominates over power corrections. Note that we are expanding in  $m_M/E_M$  so one might expect the predictions to get worse for heavier states. For the case of color suppressed decays to light mesons that are not isosinglets an analogous result was obtained in Ref. [6]. It was shown that the long distance contribution vanishes for  $M = \pi, \rho$ , so no restriction to longitudinal polarization is required for  $M = \rho$ , but a restriction is needed for  $M = K^*$ .



Thus, for these color suppressed decays SCET predicts

$$\begin{aligned} \frac{Br(\bar{B} \rightarrow D^* \pi^0)}{Br(\bar{B} \rightarrow D \pi^0)} &= \frac{Br(\bar{B} \rightarrow D^* \rho^0)}{Br(\bar{B} \rightarrow D \rho^0)} = \frac{Br(\bar{B} \rightarrow D^* \bar{K}^0)}{Br(\bar{B} \rightarrow D \bar{K}^0)} = \frac{Br(\bar{B} \rightarrow D^* \bar{K}_{\parallel}^{*0})}{Br(\bar{B} \rightarrow D \bar{K}^{*0})} = 1, \\ \frac{Br(\bar{B} \rightarrow D_s^{*+} K^-)}{Br(\bar{B} \rightarrow D_s^+ K^-)} &= \frac{Br(\bar{B} \rightarrow D_s^{*+} K_{\parallel}^{*-})}{Br(\bar{B} \rightarrow D_s^+ K^{*-})} = 1. \end{aligned} \quad (21)$$

The factorization proven with SCET for color allowed decays [5] also predicts the equality of the  $D$  and  $D^*$  branching fractions [10].

Fig. (3) summarizes the heavy quark symmetry predictions for cases where data is available. We show the ratio of amplitudes because our power expansion was for the amplitudes making it easier to estimate the uncertainty. There is remarkable agreement in the color allowed channel where the error bars are smaller and good agreement in the color suppressed channels as well.

So far our parameterization of the mixing between isosinglets in the factorization theorem has been kept completely general, and we have not used the known experimental mixing properties of  $\eta$ - $\eta'$  and  $\phi$ - $\omega$ . For the next set of predictions we use the flavor structure of the SCET<sub>II</sub> operators and the isosinglet mixing properties to a) relate the  $\eta$  and  $\eta'$  channels and b) show that decays to  $\phi$ 's are suppressed. Our discussion of mixing parameters follows that in Refs. [26, 27, 28, 29]. In general for a given isospin symmetric basis there are two light quark operators and two states (say  $\eta$  and  $\eta'$ ) so there are four independent decay constants. These can be traded for two decay constants and two mixing angles. In an SU(3) motivated singlet/octet operator basis,  $\{(\bar{u}u + \bar{d}d + \bar{s}s)/\sqrt{3}, (\bar{u}u + \bar{d}d - 2\bar{s}s)/\sqrt{6}\}$ , we have

$$f_1^\eta = -f_1 \sin \theta_1, \quad f_1^{\eta'} = f_1 \cos \theta_1, \quad f_8^\eta = f_8 \cos \theta_8, \quad f_8^{\eta'} = f_8 \sin \theta_8. \quad (22)$$

An alternative is the flavor basis used in Eq. (3),  $\{O_q, O_s\} \sim \{(\bar{u}u + \bar{d}d)/\sqrt{2}, \bar{s}s\}$ . Here

$$f_q^\eta = f_q \cos \theta_q, \quad f_q^{\eta'} = f_q \sin \theta_q, \quad f_s^\eta = -f_s \sin \theta_s, \quad f_s^{\eta'} = f_s \cos \theta_s. \quad (23)$$

Phenomenologically,  $(\theta_8 - \theta_1)/(\theta_8 + \theta_1) \simeq 0.4$  which can be attributed to sizeable SU(3) violating effects, whereas  $(\theta_q - \theta_s)/(\theta_q + \theta_s) \lesssim 0.06$  where a non-zero value would be due to OZI violating effects [21]. We therefore adopt the FKS mixing scheme [21, 29] where OZI violating effects are neglected and the mixing is solely due to the anomaly. Here one finds experimentally

$$\theta_q \simeq \theta_s \simeq \theta = 39.3^\circ \pm 1.0^\circ. \quad (24)$$

Thus it is useful to introduce the approximately orthogonal linear combinations

$$|\eta_q\rangle = \cos \theta |\eta\rangle + \sin \theta |\eta'\rangle, \quad |\eta_s\rangle = -\sin \theta |\eta\rangle + \cos \theta |\eta'\rangle, \quad (25)$$

since neglecting OZI effects the offdiagonal terms  $\langle 0|O_q|\eta_s\rangle$  and  $\langle 0|O_q|\eta_s\rangle$  are zero. Since this is true regardless of whether these operators are local or non-local, the matrix elements in Eqs. (8,18) must obey the same pattern of mixing as in Eq. (23) [ $f_q^\eta \phi_q^\eta(x) = f_q \phi_q(x) \cos \theta_q$ , etc.] and so

$$\phi_q^\eta(x) = \phi_q^{\eta'}(x) = \phi_q(x), \quad \phi_s^\eta(x) = \phi_s^{\eta'}(x) = \phi_s(x), \quad \Psi_\eta^{(0,8)} = \Psi_{\eta'}^{(0,8)} = \Psi_q^{(0,8)}. \quad (26)$$

The SCET<sub>II</sub> operators of Eq. (12) which contribute to  $A_{glue}^{(*)M}$  can produce both the  $\eta_q$  and  $\eta_s$  components of the isosinglet mesons. However, recall that at LO in  $\alpha_s(\sqrt{E\Lambda})$  the convolution over the momentum fractions in  $A_{glue}^{(*)M}$  vanishes allowing us to ignore this contribution. The remaining contributions from  $A_{short}^{(*)M}$  and  $A_{long}^{(*)M}$  involve operators that can only produce the  $\eta_q$  component of the isosinglet mesons as seen by the flavor structure of the operators in Eqs. (6) and (18). We can now write the amplitude for the  $\eta^{(\prime)}$  channels in the form

$$A^{(*)\eta} = \cos\theta [A_{short}^{(*)\eta_q} + A_{long}^{(*)\eta_q}], \quad A^{(*)\eta'} = \sin\theta [A_{short}^{(*)\eta_q} + A_{long}^{(*)\eta_q}]. \quad (27)$$

This leads to a prediction for the relative rates with SCET

$$\frac{Br(\bar{B} \rightarrow D\eta')}{Br(\bar{B} \rightarrow D\eta)} = \frac{Br(\bar{B} \rightarrow D^*\eta')}{Br(\bar{B} \rightarrow D^*\eta)} = \tan^2(\theta) = 0.67, \quad (28)$$

with uncertainties from  $\alpha_s(\sqrt{E\Lambda})$  that could be at the  $\sim 35\%$  level. Experimentally the results in Table I imply

$$\frac{Br(\bar{B} \rightarrow D\eta')}{Br(\bar{B} \rightarrow D\eta)} = 0.61 \pm 0.12, \quad \frac{Br(\bar{B} \rightarrow D^*\eta')}{Br(\bar{B} \rightarrow D^*\eta)} = 0.51 \pm 0.18, \quad (29)$$

which agree with Eq. (28) within the  $1\text{-}\sigma$  uncertainties.

For the isosinglet vector mesons we adopt maximal mixing which is a very good approximation (meaning minimal mixing in the FKS basis), and is consistent with the anomaly having a minimal effect on these states and with neglecting OZI effects. In this case only  $\langle 0|O_q|\omega\rangle$  and  $\langle 0|O_s|\phi\rangle$  are non-zero. Thus only  $A_{short}^{(*)\omega}$  and  $A_{long}^{(*)\omega}$  are non-zero and we predict that  $\phi$  production is suppressed

$$\frac{Br(\bar{B}^0 \rightarrow D^{(*)0}\phi)}{Br(\bar{B}^0 \rightarrow D^{(*)0}\omega)} = \mathcal{O}\left(\alpha_s^2(\sqrt{E\Lambda}), \alpha_s(\sqrt{E\Lambda})\frac{\Lambda_{\text{QCD}}}{Q}, \frac{\Lambda_{\text{QCD}}^2}{Q^2}\right) \lesssim 0.2, \quad (30)$$

possibly explaining why it has not yet been observed. Interestingly a measurement of  $\bar{B} \rightarrow D\phi$  or  $\bar{B} \rightarrow D^*\phi$  may give us a direct handle on the size of these expansion parameters.

Just using the original form of the electroweak Hamiltonian in Eq. (1) there is an SU(3) flavor symmetry relation among the color suppressed decays [30]

$$R_{\text{SU}(3)} = \frac{Br(\bar{B}^0 \rightarrow D_s^+ K^-)}{Br(\bar{B} \rightarrow D^0 \pi^0)} + \left| \frac{V_{ud}}{V_{us}} \right|^2 \frac{Br(\bar{B}^0 \rightarrow D^0 \bar{K}^0)}{Br(\bar{B} \rightarrow D^0 \pi^0)} - \frac{3Br(\bar{B}^0 \rightarrow D^0 \eta_8)}{Br(\bar{B} \rightarrow D^0 \pi^0)} = 1, \quad (31)$$

$$R_{\text{SU}(3)}^* = \frac{Br(\bar{B}^0 \rightarrow D_s^{*+} K^-)}{Br(\bar{B} \rightarrow D^{*0} \pi^0)} + \left| \frac{V_{ud}}{V_{us}} \right|^2 \frac{Br(\bar{B}^0 \rightarrow D^{*0} \bar{K}^0)}{Br(\bar{B} \rightarrow D^{*0} \pi^0)} - \frac{3Br(\bar{B}^0 \rightarrow D^{*0} \eta_8)}{Br(\bar{B} \rightarrow D^{*0} \pi^0)} = 1,$$

where  $\eta_8$  is the SU(3) octet component of the  $\eta$ . In the SU(3) limit the  $\eta - \eta'$  mixing vanishes and we can take  $\eta_8 = \eta$ . Away from this limit there is SU(3) violation from the mixing as well as from other sources, and it is the latter that we would like to study. To get an idea about the effect of mixing we set  $|\eta_8\rangle = \cos\vartheta|\eta\rangle + \sin\vartheta|\eta'\rangle$ , which from Eq. (25) can then be written in terms of  $|\eta_q\rangle$  and  $|\eta_s\rangle$ , and vary  $\vartheta$  between  $-10^\circ$  and  $-23^\circ$ . From the flavor structure of the leading order SCET operators for  $B \rightarrow DM$  decays we then find

$$\frac{Br(\bar{B}^0 \rightarrow D\eta_8)}{Br(\bar{B}^0 \rightarrow D\eta)} = \frac{Br(\bar{B}^0 \rightarrow D^*\eta_8)}{Br(\bar{B}^0 \rightarrow D^*\eta)} = \frac{\cos^2(\theta - \vartheta)}{\cos^2(\theta)}, \quad (32)$$

where  $\vartheta$  is the  $\eta$ - $\eta'$  state mixing angle in the flavor octet-singlet basis and  $\theta$  is the FKS mixing angle. In the SU(3) limit  $\vartheta = \theta_1 = \theta_8 = 0$ , however phenomenologically  $\vartheta \simeq -10^\circ$  to  $-23^\circ$ . Experimentally taking  $|V_{us}/V_{ud}| = 0.226$  and using Table I gives

$$R_{\text{SU}(3)} = \begin{cases} 1.00 \pm 0.59 & [\vartheta = 0^\circ] \\ 1.75 \pm 0.57 & [\vartheta = -10^\circ] \\ 2.64 \pm 0.56 & [\vartheta = -23^\circ] \end{cases}, \quad R_{\text{SU}(3)}^* = \begin{cases} -0.22 \pm 0.97 & [\vartheta = 0^\circ] \\ 0.59 \pm 0.88 & [\vartheta = -10^\circ] \\ 1.57 \pm 0.83 & [\vartheta = -23^\circ] \end{cases}. \quad (33)$$

In all but one case the central values indicate large SU(3) violation, however the experimental uncertainty is still large. It would be interesting to compute the uncertainties by properly accounting for correlations between the data rather than assuming these correlations are zero as we have done. At  $1\text{-}\sigma$  the errors accommodate  $R_{\text{SU}(3)}^* = 1$  except if  $\vartheta = 0^\circ$ , and only accommodate  $R_{\text{SU}(3)} = 1$  if  $\vartheta = 0^\circ$ . Note that the heavy quark symmetry prediction,  $R_{\text{SU}(3)}^* = R_{\text{SU}(3)}$ , is still accommodated within the error bars.

In the pQCD approach predictions for color suppressed decays to isosinglets have been given in Refs. [31, 32], where they treat the charm as light and expand in  $m_c/m_b$ . With such an expansion there is no reason to expect simple relationships between decays to  $D$  and  $D^*$  mesons because heavy quark symmetry requires a heavy charm. In Ref. [32] predictions for  $\eta$  and  $\eta'$  were given dropping possible gluon contributions. Our analysis shows that this is justified and predicts a simple relationship between these decays, given above in Eq. (28).

To conclude, we derived a factorization theorem which describes color suppressed decays to isosinglets solely from QCD without model dependent assumptions by expanding in  $\Lambda_{\text{QCD}}/Q$ . Phenomenological implications were discussed for  $B \rightarrow D\eta$ ,  $D\eta'$ ,  $D\omega$ ,  $D\phi$ . We proved that the gluon production amplitude involves the same soft  $B \rightarrow D$  matrix element as the non-gluon terms. We then showed that the factorized form of the amplitudes together with heavy quark symmetry predict that  $Br(\bar{B} \rightarrow D^*\{\eta, \eta', \omega_{||}, \phi_{||}\}) = Br(\bar{B} \rightarrow D\{\eta, \eta', \omega, \phi\})$ , with corrections being suppressed by either a power  $\Lambda_{\text{QCD}}/Q$  or a factor of  $\alpha_s(Q)$ . The  $\alpha_s(Q)$  terms can be computed in the future. We also consider  $\eta$ - $\eta'$  mixing and showed that due to the vanishing of the gluon contributions the flavor structure of the SCET operators imply  $Br(\bar{B} \rightarrow D^{(*)}\eta')/Br(\bar{B} \rightarrow D^{(*)}\eta) = \tan^2(\theta) = 0.67$  where  $\theta = 39.3^\circ$  is the  $\eta$ - $\eta'$  mixing angle in the FKS scheme, and that  $Br(\bar{B} \rightarrow D^{(*)}\phi)/Br(\bar{B} \rightarrow D^{(*)}\omega) \lesssim 0.2$ . Corrections here are only order  $\alpha_s(\sqrt{E\Lambda})$  and should be computed in the near future. At one-loop the effect of operator mixing will also need to be considered [33]. Finally, tests of SU(3) symmetry were given in Eqs. (31-33).

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